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Methods of Travel-Time Residual Declustering for the Knowledge Base Calibration and Integration Tool (KBCIT)

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Abstract

Calibration of seismic nuclear test monitoring stations relies on a diverse reference-event database. The reference-event covariance structure must be characterized and the covariance propagated to subsequent processes and calibration products. In seismic location one of the first steps in the calibration process is declustering, in which closely spaced epicenters are combined to reduce redundant data and random observational errors. We formulate a new declustering procedure that accounts for correlated and uncorrelated components of reference-event error, producing a declustered data set that tracks raw reference-event uncertainties. Declustering behavior is demonstrated using example data sets.

Introduction

The Department of Energy (DOE) Ground-Based Nuclear Explosion Monitoring (GNEM) program is actively integrating a collection of seismic reference events for use in development and validation of earth models and empirical travel time correction surfaces. The result of this effort is a collection of events with varying degrees of location accuracy and inter-event correlation. Assessment of the reference-event covariance structure and methods to propagate this covariance structure through the calibration and validation procedures are critical to the DOE Knowledge Base (KB) effort. Although development of methods to characterize the covariance structure is an ongoing topic of research (e.g. Sweeney, 1998; Myers and Schultz, 2001), we continue to develop calibration procedures that utilize a general reference-event covariance structure.

One of the most powerful applications of the reference-event catalog is the development of empirical, travel-time correction surfaces using Bayesian kriging (Schultz et al., 1998; Myers and Schultz, 2000). Continuous correction surfaces are developed by combining travel-time residuals from geographically distributed reference events using a statistically rigorous algorithm (kriging) that works to extract correlated model error information while tracking errors that result from other random processes.

Event declustering is a common pre-processing step for kriging. Declustering reduces closely spaced events to a single point, producing an average value in a more computationally efficient algorithm than kriging. Furthermore, declustering diminishes potential numerical instabilities in the kriging algorithm that can result from highly correlated (closely spaced) data (see Myers and Schultz, 2000). Simple declustering algorithms (e.g. Myers and Schultz, 2000) bin events geographically, average events within each bin, and calculate a post-averaging uncertainty for each declustered point.

Lawrence Livermore National Laboratory (LLNL) is working with Sandia National Laboratory (SNL) to develop a software package – the KB Calibration and Integration Tool (KBCIT) – that standardizes and streamlines the process of reducing reference event information into correction surfaces (Dodge et al, 2000). Reference event declustering is an integral part of KBCIT, and this report documents the motivation and theoretical development for the declustering algorithm used in KBCIT.

Spatial Correlation of Model Errors

The fundamental observation enabling the development of travel-time correction surfaces is the spatial correlation of travel-time residuals as determined for a single station (See Myers and Schultz, 2000 for examples and other references). For a single station, there is considerable overlap in the geologic structure sampled by seismic rays from events that are close to one another. Therefore, the ray velocity and travel-time are correlated. As inter-event distance increases, the volume of influencing geologic structure shared by the two rays becomes smaller, and correlation in travel-time residuals decreases. The shared geologic structure sampled by rays from neighboring events, combined with spatial correlation in the geologic structure, creates spatial correlation of travel-time residuals.

The goal of declustering is to minimize the number of samples (epicenters with associated travel-time residuals) while maintaining resolution of the spatial correlation structure. Therefore, it is desirable to decluster using a maximum bin size, while maintaining a high degree of correlation within the bin. When velocity-model induced correlation is high within the bin, averaging reduces uncorrelated errors and leaves the interaction of points with varying degrees of model error correlation to the kriging algorithm.

Generality of the declustering algorithm requires that we allow for varying degrees of correlation between residuals within a decluster bin. If the decluster bin size captures points with appreciably reduced model-error correlation, then the process of finding the uncertainty of the average is effected. Because the model-error is deterministic, averaging of points will not reduce the model-error component of the random processes.

Reference-Event Covariance Structure

Location errors for closely spaced reference events, whose epicenters are determined seismically, are likely to be correlated. The correlation stems from the likelihood that the seismic network for the two events are similar, so systematic errors in travel-time prediction (and thus residuals) are correlated. Of course the locating network is not constant through time and other variables, like event magnitude, may effect the constellation of detecting stations. Therefore, event proximity is not a full proof criteria for establishing location error correlation, and even events that are co-located may not have perfectly correlated errors.

We provide a hypothetical example for clarity. Our example is an area of ongoing seismic activity where an aftershock sequence has been recorded on a dense, temporary network. Notwithstanding bootstrapping relative location techniques, events within the area can be separated into two groups: 1) those events occurring during the deployment and 2) those occurring before or after the deployment. Group (1) events are likely to have smaller location errors than group (2) events. Furthermore, correlation of location error is likely to be distinct within the two groups, and there may be a level correlation between the two groups. This is just a hypothetical case and we leave the issue of establishing the covariance structure for clustered events to other studies. However, we conclude from our example that within a set of closely spaced events there may be several "groups" of events, with definable inter and intra group correlation.

Correlation in location error is likely to be a function of the distance between the events. Because correlation in reference-event error is effected by model error at all stations of the locating network (we have already discussed the distance dependence of model errors), a correlation in reference event error seems likely. However, correlation distance for reference-event error is likely to be shorter than for model errors, because location errors are dependent on model errors from a number of stations. Therefore, we are likely to observe significant changes in reference-event correlation across a decluster bin, and we must account for this distance dependance. Unlike model error correlation – which inflates the uncertainty of declustered points when the bin is large, a larger bin size helps to average out reference-event location errors, because correlation in locations is reduced with increasing separation of epicenters.

Converting Spatial Error to Travel-Time Error

Efforts to characterize the reference-event covariance structure make use of location error as a metric. In the case of travel-time correction, we are interested in the covariance of travel-time residuals resulting from the covariance of location errors, which requires mapping errors through the appropriate travel-time model. Using conventional statistical techniques we arrive at:

$$\sigma_{tt}^2 = \left(\frac{\partial t}{\partial \Delta}\sigma_{dist}\right)^2 \qquad [1]$$

where σ^2_{tt} is the travel-time variance resulting from event location uncertainty, σ_{dist}. The term $\partial t/\partial \Delta$ is the partial derivative of travel-time with respect to distance, evaluated at the appropriate event/station distance. We note that this formulation is a conservative estimate that assumes event error is along the great-circle path. We may include a factor to account for random orientation of the event mislocation vector.

Figure 1 illustrates the distance-dependant effect that event mislocation has on the travel-time residual uncertainty. Note that within teleseismic distance (> ~30°) and regional distance (~1.5°<distance< ~13°) event mislocation effects travel-time residuals approximately equally (equally within each distance range, not between the two distance ranges). At upper-mantle distances (~13°<distance< ~30°) and local distance (> ~1.5°) travel-time residuals resulting from event mislocation are appreciably non-stationary.

The KBCIT Decluster Method

We now include issues discussed above into the KBCIT event declustering algorithm. The travel-time residual can be broken down into three parts.

$$t_{res} = t_p + t_{re} + t_m \qquad [2]$$

The terms t_{res},t_p,t_{re}, and t_m are travel-time residual and error due to picking, event mislocation, and model inaccuracies, respectively. Expanding the expected value of the squared residual, we find that:

$$\sigma^2_{res} = \sigma^2_p + \sigma^2_{re} + \sigma^2_m + 2\left[C(p,re) + C(p,m) + C(re,m)\right] \quad [3]$$

where σ^2 is the variance and C denotes the covariance.

For KBCIT we assume that covariance terms in Eqn [3] can be neglected. Assuming the picking error is not correlated with other variables, the first two covariance terms of Eqn [3] are zero. We may also eliminate the covariance between reference-event error and model error in some instances. If, for example, a teleseismic network is used to locate the reference event, then the reference-event error is not likely to be correlated with model error for regional stations. Additionally, if the reference event is located with numerous, geographically distributed stations, then the reference-event error is less likely to be correlated with model error at any given station.

Establishing that travel-time residuals can be reduced to three independent error processes and events in the each decluster bin are likely to belong to different groups, we can proceed with the declustering algorithm. First we find the expected value for each group of events within the bin. This is accomplished by averaging travel-time residuals. In the case of KBCIT we have the option of finding the mean or median. The variance of each event group accounts for potential correlation in errors.

$$\tau^2_{re_i} = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{k=1}^{N} C_{jk} \quad [4]$$

where τ^2 is the variance of the mean, N is the number of events, C_{jk} is the covariance, j and k count the events in the group. For a typical pair of reference events belonging to the same group, the C_{jk} matrix has the form:

$$C = \begin{bmatrix} \sigma^2_{p1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{p2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{re1} & Cov(re_1, re_2) & 0 & 0 \\ 0 & 0 & Cov(re_1, re_2) & \sigma^2_{re2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{m1} & \gamma_{m1,m2} \\ 0 & 0 & 0 & 0 & \gamma_{m1,m2} & \sigma^2_{m2} \end{bmatrix} \quad [5]$$

where γ is the variogram value (with picking error removed) appropriate for the two points, and $Cov(re_1,re_2)$ is the covariance of travel-time residuals based on the model-error correlation.

After averaging the events belonging to individual groups, we combine the groups into a single decluster value via a weighted sum.

$$t_{decluster} = \sum_{i=1}^{M} w_i \bar{t}_{res_i} \qquad [6]$$

where $t_{decluster}$ is the declustered value, w_i is the weight of the i_{th} group, and t_{res} is the group average. The weights are a normalized inverse of the decluster variance.

$$w_i = \frac{1}{\tau_i^2} \frac{1}{1/\sum_{j=1}^{M} 1/\tau_j^2} \qquad [7]$$

The variance of the declustered point is then:

$$\tau_{decluster}^2 = \sum_{i=1}^{M} \left(w_i \tau_i\right)^2 \qquad [8]$$

Discussion and Conclusions

Figure 2 is an example of the reduction in redundant data achieved by declustering. In this example the bin size is 0.25°, so correlation in model error over the bin is nearly perfect. Even with this relatively small bin size the number of data are reduced by about three times. Because kriging requires the inversion of a matrix with dimensions that are the determined by the number of data and computational cost of matrix inversion scales as the square of matrix size, we estimate that declustering results in a nine-fold speed up. Also note that the spatial data coverage is not significantly effected by declustering. Therefore, resolution of correlated model error is not significantly effected.

Figure 3 illustrates how different RE and picking error levels combine in the estimation of the declustered point. The number following RE indicate the 95% confidence in the event location error (km), so two RE15 events are combined with one RE2 event. When picking error is equally high (1.0 second) – relative to the magnitude of the residual – for each event, picking error is the dominant error process. Therefore, the declustered value is close to the mean. When picking error is uniformly reduced to 0.1 second, RE error becomes more important in the averaging process, and the declustered value moves towards the residual value for the RE2 event. When picking error is 1.0 second for the RE15 events and 0.1 second for the RE2 event, the declustered value us further shifted towards the RE2 value. Declustering accounts for both RE location error and random picking error. Inflation of either error term – relative to other events in the bin – results in down weighting.

The final example of the KBCIT declustering routine demonstrates the effect of increasing the bin size. Although larger bins are likely to capture more points, declustering by averaging does not asymptotically decrease uncertainty to zero as the number of points increases. Rather, the uncertainty asymptotically approaches the variogram value appropriate for the bin size. Figure 4 shows the uncertainty of the declustered value for bin sizes of 1° and 3.5° (stars and pluses, respectively). The example in Figure 4 has a spatially correlated error function with a range of 5° -- so points that are more then 5° apart are uncorrelated. When the bin size is 1° the spatial correlation for points within the bin is at least 0.85, and the dominant error process is uncorrelated. Therefore, simple averaging significantly reduces the uncertainty of declustered point. The other extreme is the large bin size where the correlated process significantly adds to the bin variance, diminishing the effectiveness of averaging.

The examples presented here focus on travel-time residuals, but there is considerable flexibility in the declustering process and application can be tailored to other uses. These routines can be used on any spatially distributed random process that can be separated into correlated and correlated components. We add the further utility of mapping uncertainty processes through a linear function.

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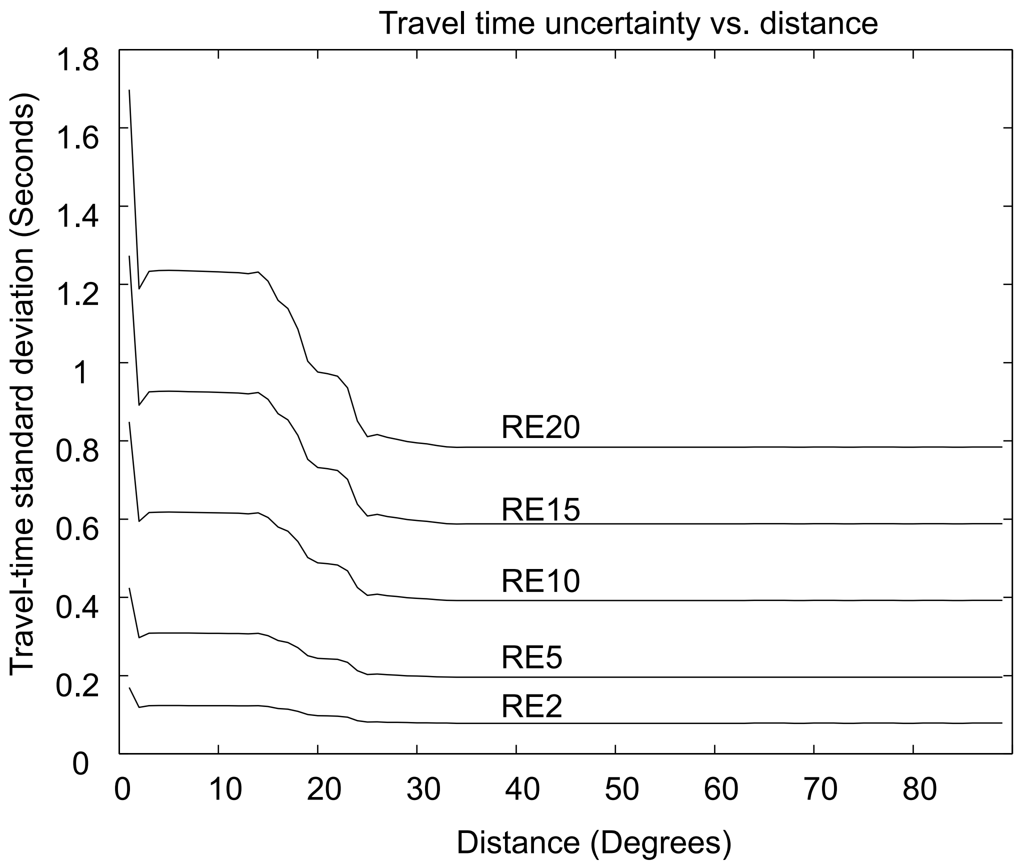

Figure 1. Travel-time uncertainty resulting from epicenter uncertainty as a function of distance for iasp91 P-waves is shown. The epicenter uncertainty is a worst-case scenario, where the error occurs along the great-circle path. Each curve is labeled by reference event (RE) level, where the number is the 95% confidence in the location accuracy (km). Note that overshoot at changes in gradient of the curves is the result of errant spline interpolation and should be ignored.

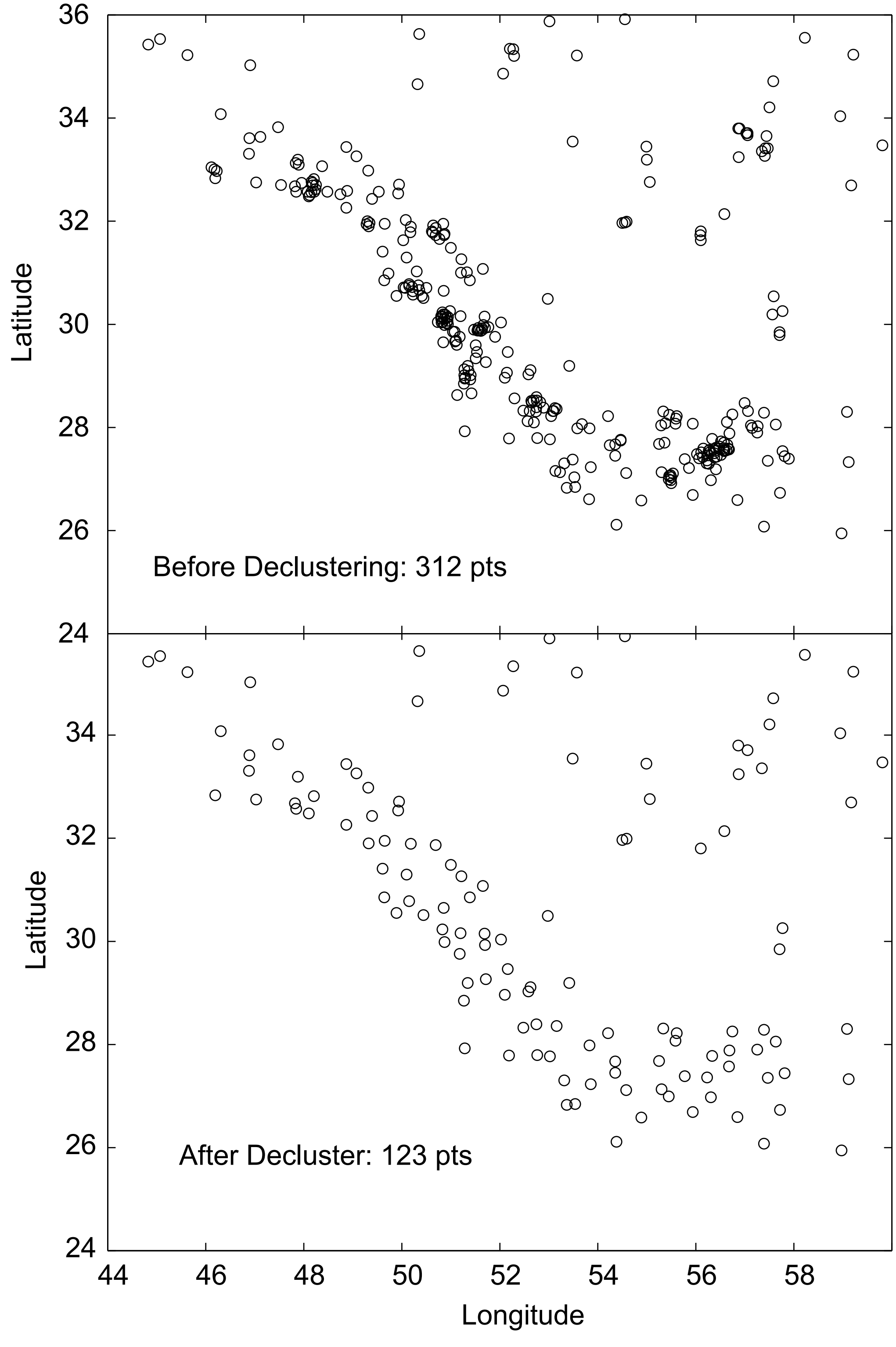

Figure 2. Declustering with 0.25 degree bin size reduces the number of Zagros epicenters observed at stations NIL by about a factor of 3. Note that before declustering many of the events are tightly clustered, resulting in redundant information about the model error at the cluster location. After declustering the spatial samping is preserved, allowing the spatial correlation of model error to be captured with a much smaller data set. Computational cost in kriging is estimated by squaring the number of points, so we expect kriging to be about 9 times faster after declustering.

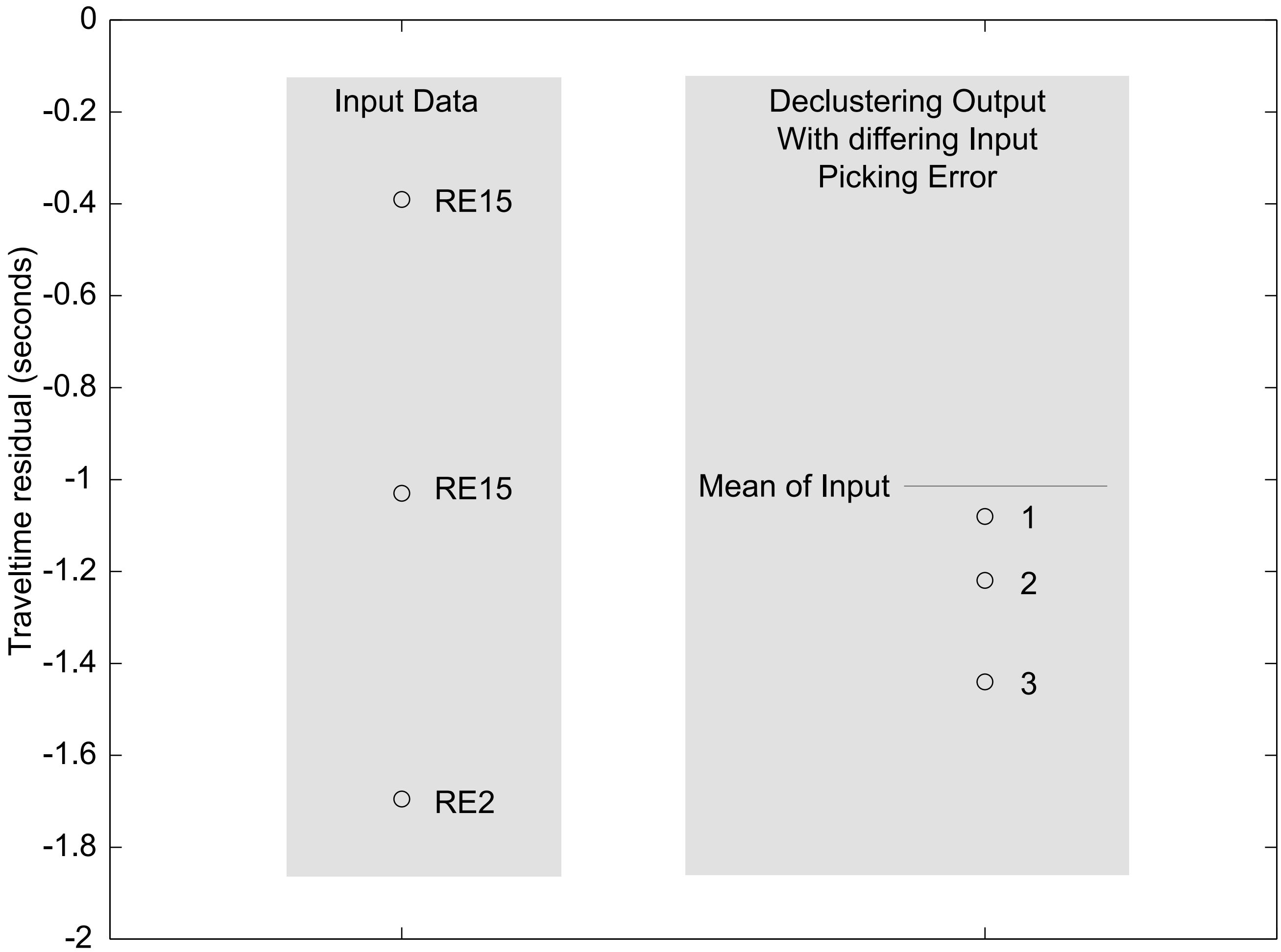

Figure 3 The effect of reference event (RE) level and picking error on the declustered value are illustrated for the events in the Gulf of Aqaba vicinity observed at station NIL. Output 1 results when picking error for each input event is 1.0 second. For output 2 the picking error is 0.1 second for each event, and for output 3 the picking error is 1.0 second for the RE15 events and 0.1 second for the RE2 event. See text for discussion.

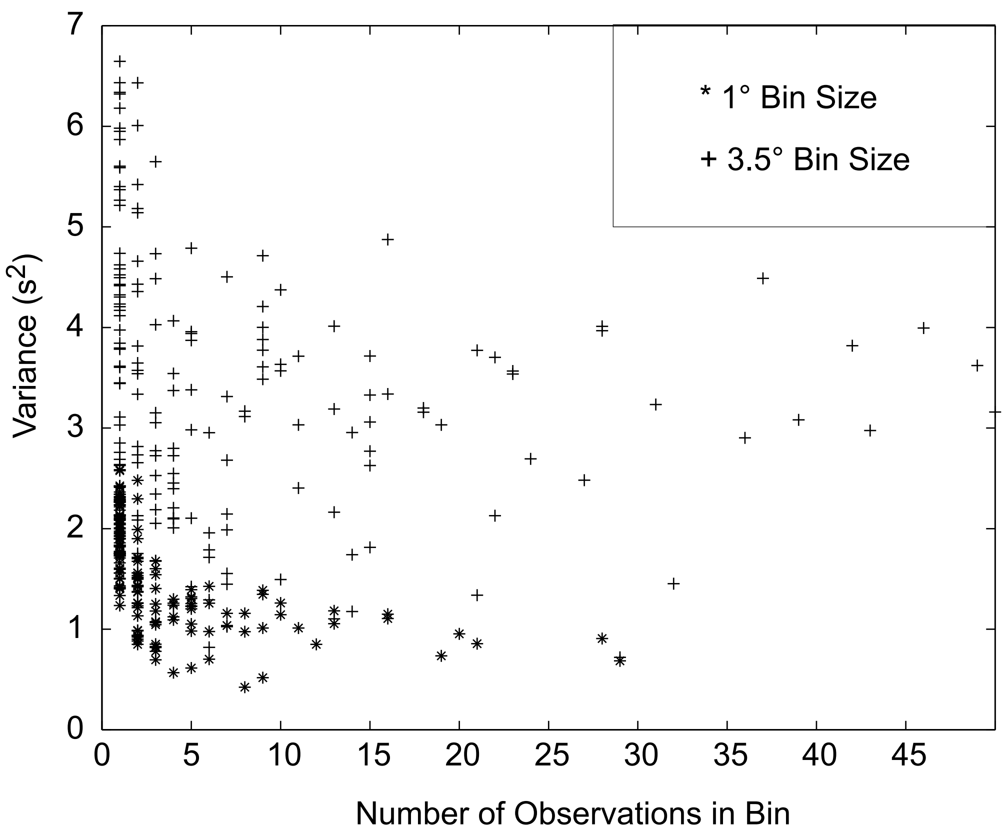

Figure 4. The uncertainty of declustered values changes with bin size and the number of points in the bin. For a 1° bin size (stars) the uncertainty of the declustered value rapidly approaches a value of about 1 second, which is the picking error of this data set. For a 3.5° bin size the declustered uncertainty is higher because of contributions from the correlated error component.